Reg. No. :

Question Paper Code : X 67615

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2020
Second Semester
Civil Engineering
MA 1151 – MATHEMATICS – II
(Common to all Branches)
(Regulations 2008)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions.

PART – A **(10×2=20 Marks)**

1. Find the Laplace transform of $\frac{\sin at}{t}$.

2. Find the inverse Laplace transform of $\frac{s+1}{s^2-2s}$.

3. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then find $\nabla.\hat{r}$ and $\nabla \times \hat{r}$

4. Find the directional derivation of $f(x, y, z) = xy^2 + yz^2$ at the point (2, –1, –1) in the direction of the vector $\hat{i} + 2\hat{j} - 2\hat{k}$

5. State whether or not $f(z) = \bar{z}$ is an analytic function.

6. Find the image of the circle $|z| = 2$ under the transformation $w = 3z$.

7. Evaluate $\int_0^2 \int_0^x (x+y)\,dx\,dy$.

8. Evaluate $\iint e^{-x-y}\,dxdy$ over R, where R is the region in the first quadrant in which $x + y \leq 1$.

9. Evaluate $\int_C \frac{z^2+1}{z^2-1}dz$, where C is a circle of unit radius and centre at $z = i$.

10. Find the Laurent's series for the function $f(z) = z^2 e^{1/z}$ about $z = 0$.

PART – B (5×16=80 Marks)

11. a) i) Find the Laplace transform of the following functions

1) $t^2 e^{-1} \cos t$

2) $\dfrac{e^{at} - \cos bt}{t}$. (8)

ii) Using convolution theorem, find $L^{-1}\left\{\dfrac{1}{\left(s^2+a^2\right)^2}\right\}$. (8)

(OR)

b) i) Find the Laplace transform of $f(t) = \begin{cases} t, & 0 \le t \le a \\ 2a - t, & a \le t \le 2a \end{cases}$ $f(t + 2a) = f(t)$. (8)

ii) Using Laplace transform, solve $\dfrac{d^2y}{dt^2} + y = \sin 2t$, with $y(0) = 0$ and $y'(0) = 0$. (8)

12. a) i) Evaluate $\iint_S \vec{F}.dS$, where $\vec{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$ and S is the part of the sphere $x^2 + y^2 + z^2 = 1$ that lies in the first octant. (8)

ii) Verify Stoke's theorem for $\vec{F} = y\hat{i} + 2yz\hat{j} + y^2\hat{k}$, where S is the upper half of the sphere $x^2 + y^2 + z^2 = a^2$ and C is the circular boundary on the XOY plane. (8)

(OR)

b) i) Verify Green's theorem in a plane with respect to $\int_C \left(2x^2 - y^2\right)dx + \left(x^2 + y^2\right)dy$, where C is the boundary of the region in the XOY-plane enclosed by the x-axis and the upper half of the circle $x^2 + y^2 = 1$. (8)

ii) Find the work done by the force $\vec{F} = \left(x^2 + y^2\right)\hat{i} + \left(x^2 + z^2\right)\hat{j} + y\hat{k}$, when it moves a particle along the upper half of the circle $x^2 + y^2 = 1$ from the point (–1, 0) to the point (1, 0). (8)

13. a) i) Determine the analytic function $f(z) = u + iv$ given that $u = e^x (x \cos y - y \sin y)$ (8)

ii) Find the bi-linear transformation which maps the points (–1, 0, 1) of the z-plane into the points (0 i, 3i) of the w-plane. (8)

(OR)

b) i) If $w = f(z)$ is an analytic function of z, prove that $\nabla^2 |f(z)|^2 = 4|f'(z)|^2$. (8)

ii) Find the image of the circle $|z - 3i| = 3$ under the transformation $w = 1/z$. (8)

14. a) i) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx\,dy\,dz}{\sqrt{1-x^2-y^2-z^2}}$. **(8)**

ii) Find by double integration the area between the two parabolas $3y^2 = 25x$ and $5x^2 = 9y$. **(8)**

(OR)

b) i) Evaluate by changing to polar coordinates, the integral **(8)**

$\int_0^a \int_0^{\sqrt{a^2-x^2}} (x^2y + y^3)\,dx\,dy$.

ii) Evaluate $\iiint_R \frac{1}{x^2+y^2+z^2}\,dxdydz$ throughout the volume of the sphere

$x^2 + y^2 + z^2 = a^2$. **(8)**

15. a) i) Using Cauchy's integral formula, evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-2)(z-3)}\,dz$, where C is the circle $|z| = 4$. **(8)**

ii) Evaluate, using contour integration, $\int_0^{2\pi} \frac{d\theta}{1 - 2p\cos\theta + p^2}$, $0 < p < 1$. **(8)**

(OR)

b) i) Find the Laurent's series of $f(z) = \frac{1}{z(1-z)}$ valid in the regions $1 < |z + 1| < 2$ and $|z + 1| > 2$. **(8)**

ii) Evaluate $\int_{-\infty}^{\infty} \frac{x^2\,dx}{(x^2+a^2)(x^2+b^2)}$, using contour integration where $a > b > 0$. **(8)**
